

Sequences for Binary Encoding of Radar Signals

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Abstract. Intra-pulse modulation with Walsh-Hadamard sequences (WHS) provides a mean of synthesizing large orthogonal pulse radar signal constellations. In this paper the performance of WHS for binary encoding of radar signals is described. It is shown that between WHS with length 8, 16 and 32 it is possible to find a set of orthogonal sequences with good properties. Probability of correct range detection of selected WHS in presence of active noise jamming is described and it is compared with cases when Barker code is used for binary encoding of radar signals and when widely used linear frequency modulation inside pulse duration is used to spread spectrum of radar signals.

Keywords: Pulse compression methods; Radar signal processing; Radar waveform.

1 Introduction

Because of their unique autocorrelation properties, some binary codes, such as Barker, Walsh-Hadamard or other sequences are often found in modern pulse radars. Generally, the radar needs a large peak signal power to average noise power ratio (signal-to-noise ratio - SNR) at the time of the target return signal for good detection performance. The maximal achievable SNR depends on the total transmitted energy, but not on the presence of any modulation. Thus for good detection, many radars seek to transmit long-duration pulses to achieve high energy, because transmitters typically operates near their peak power limitation [1]. The use of a constant-frequency waveform imposes the two serious restrictions on the shape of the ambiguity function: the product of delay and Doppler resolution remains near to unity, and the pulse energy is directly proportional to the width of the resolution element in delay. To obtain adequate SNR on small targets, it may be necessary to extend the pulse width beyond the separation delay of the nearest target, preventing them from being resolved. If phase modulation of transmitted pulse is permitted, both these restrictions can be bypassed, and the resulting process is called pulse compression [2].

Pulse compression is a radar signal processing technique to increase the range resolution as well as to improve the SNR. This is achieved by modulating the transmitted pulse (spectrum spreading) and then correlating the received signal with

the transmitted pulse. Rapid evolution of digital technologies brings also new approaches to the field of radar signal's designing and processing.

The resistance against active jamming in military radar applications is the most important factor when one compares their performance. It can be achieved by spread spectrum techniques which can be classified according to the method of spectrum spreading to Direct Sequence Spread Spectrum (DSSS), Frequency Hopping Spread Spectrum (FHSS) and Linear or Non-Linear Frequency Modulation inside pulse duration (LFM or NLFM). New types of radar signals were developed by applying spread spectrum techniques into the pulse radar technology. They are known as signals with intra-pulse modulation (IM). In this case a radar pulse is divided into multiple subpulses and frequency or phase modulation is applied inside of each subpulse. From this point of view, it is possible to classify radar signals with intra-pulse modulation into the following groups: signals with continuous (linear or nonlinear) frequency modulation, signals with discrete frequency modulation, and signals with binary phase modulation [3], [4], [5].

The main problem when using binary coded signals is finding sequences with proper autocorrelation properties. Binary sequences with low aperiodic side lobes of autocorrelation and small mutual cross correlation are required. Optimization of the polyphase orthogonal code and orthogonal discrete frequency-coding waveforms by Simulated Annealing (SA) optimization algorithms and Genetic Algorithms respectively is described in [6], [7], [8], and the optimization criterion is not only to minimize autocorrelation sidelobe peak and the cross-correlation peak but also minimize the total autocorrelation side lobe energy and cross-correlation energy. The algebraic methods to construct the polyphase sequence with good aperiodic autocorrelation and cross-correlation values and a good tolerance to small Doppler shift are proposed in [9].

Using of Barker codes of different length for binary encoded radar signals was proposed in [10], because they have good autocorrelation properties. Their disadvantage is that for defined length (7, 11, 13) there is only one sequence.

In this paper performance of Walsh-Hadamard sequences (WHS) for binary encoding of radar signals is described. It is shown that between WHS with length 8, 16 and 32 it is possible to find a set of orthogonal sequences with good properties. Probability of correct range detection of selected WHS in presence of active noise jamming is described and it is compared with cases when Barker code is used for binary encoding of radar signals and when widely used linear frequency modulation inside pulse duration is used to spread spectrum of radar signals.

2 Walsh-Hadamard sequences for binary encoded radar signals

2.1 Walsh-Hadamard Sequences

The paper proposes to use Walsh-Hadamard sequences to binary coding of radar signal - spreading the spectrum of radar pulse with BPSK intra-pulse modulation

(BPSK-IM). Walsh Hadamard sequences are easy to generate, and orthogonal in the case of perfect synchronization [11].

The Walsh-Hadamard sequences of the length N ; $N = 2^n$, $n = 1, 2, \dots$, are often defined using Hadamard matrices H_N [11], with

$$H_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \quad (1)$$

and

$$H_{2N} = \begin{bmatrix} H_N & H_N \\ H_N & -H_N \end{bmatrix}. \quad (2)$$

The resulting matrices H_N are orthogonal matrices, i.e. for every N we have:

$$H_N H_N^T = N I_N, \quad (3)$$

where H_N^T is the transposed Hadamard matrix of order N , and I_N is the $N \times N$ unity matrix.

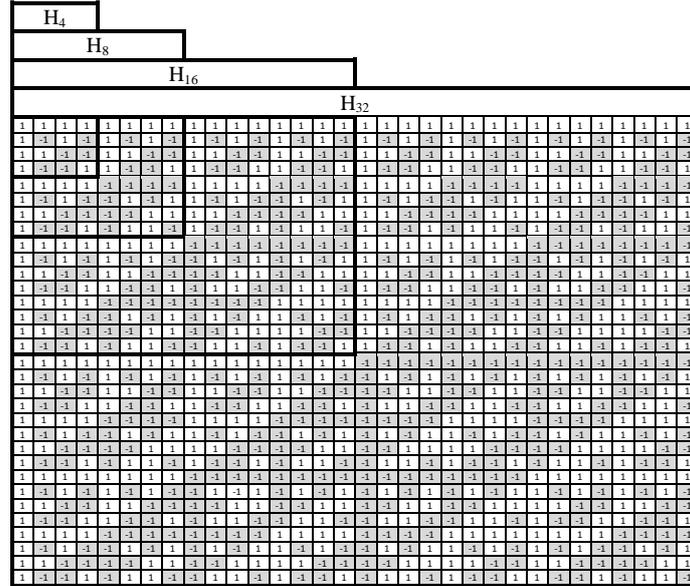


Fig. 1. Walsh-Hadamard sequences for $N = 4, 8, 16$ and 32 .

In such way we can generate N ; $N = 2^n$, $n = 1, 2, \dots$, orthogonal sequences $WHS(i, N)$, $i=1, 2, \dots, N$, with length N , each representing the i^{th} row of Hadamard matrix H_N .

In this paper H_8 , H_{16} and H_{32} are used for binary coding of radar signal and to produce BPSK-IM inside radar pulse. Walsh-Hadamard sequences (WHS) for various N are shown in Fig. 1.

2.2 Binary Encoded Radar Signals

To apply the WHS for binary coding of radar pulse, we divide the transmitted pulse of duration PW into N subpulses of duration $PW_m = PW/N$. These subpulses are used to modulate carrier frequency by BPSK modulation. Resultant signal create BPSK-IM. Generally, the radar signal with BPSK-IM in time domain may be described as [12]

$$s(t) = \begin{cases} A \cdot \sin[\omega t + \Delta\psi_m] & \text{for } t \in \langle i \cdot PRI, i \cdot PRI + PW \rangle \\ 0 & \text{for } t \in \langle i \cdot PRI + PW, i \cdot PRI + PW + DT \rangle \end{cases}, \quad (4)$$

where ω is a signal frequency, t is time, $\Delta\psi_m$ is a phase deviation in the m^{th} subpulse (value 0 when the m^{th} bit in WHS equals +1 and value π when it equals -1), PRI is the pulse repetition interval and DT is dwell time. BPSK-IM signal in time domain is shown in Fig. 2.

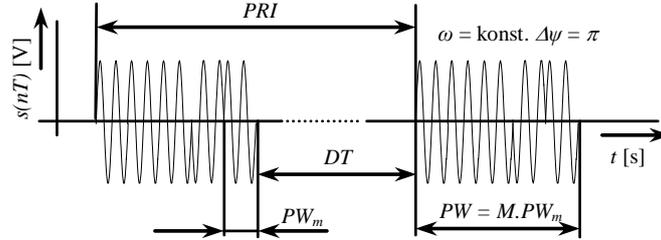


Fig. 2. BPSK intra-pulse modulated signal in time domain.

The effect of applying BPSK intra-pulse modulation with N subpulses to radar pulse is spreading its spectrum N times in frequency domain and can be represented as direct sequence spread spectrum system. At the receiver side during reception, signals which are coherent with used binary sequence are de-spread while all other signals not coherent with it are spread. So we can achieve broadband gain which is proportional to the pulse and the shortest subpulse run (PW_s) duration ratio

$$G_{BB} = 10 \log \frac{PW}{PW_s} \quad (5)$$

together with pulse compression.

2.1 Performance of Walsh-Hadamard Sequences

If we use the WHS from Fig. 1, each row of this matrix represents WHS for binary encoding of radar pulse during its pulse duration (PW). Because of WHS orthogonality it is possible to separate two radar pulses spread by two different WHSs from received signal, or in other words, to extract from received signal only that part which was spread by the certain WHS. But not all WHSs provide the same performance from the resistance to active noise jamming point of view.

Aperiodic autocorrelation function, very low or none side lobes in autocorrelation function and small mutual cross correlation are the most crucial parameters required from sequence for binary encoding of radar signals. Due to orthogonality of WHS they provide zero mutual cross correlation under condition of perfect synchronization. But not all of them provide sufficiently low side lobes in autocorrelation function. By analysis of autocorrelation functions of WHSs, the following conclusions for selection of “good sequences” can be formulated:

- Good sequence should contain so many state transition as possible;
- Good sequence should have difference between the autocorrelation function maximum and the nearest side lobes in autocorrelation function big enough; the bigger difference will produce better performance.
- Good sequence should not contain periodic pattern;
- If there is periodic pattern, sequences with longer period will provide better performance.

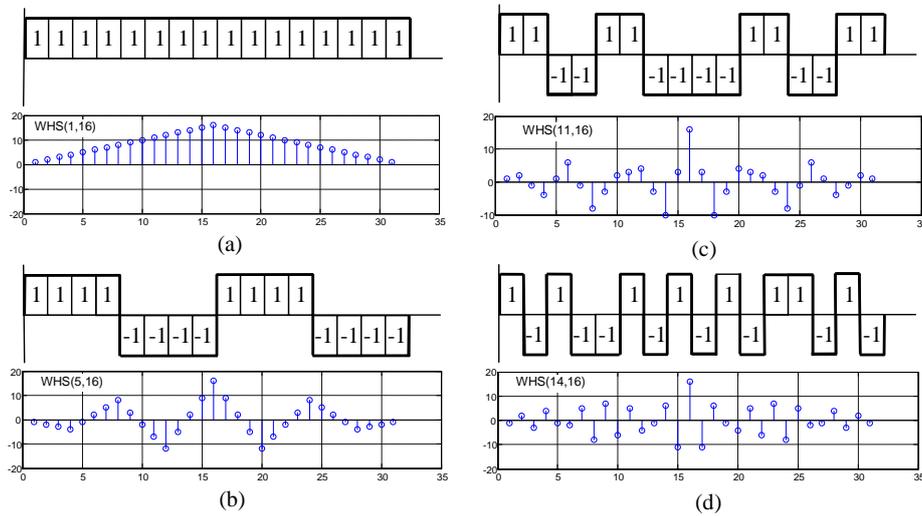


Fig. 3. Walsh-Hadamard sequences and their autocorrelation functions for $N=16$, (a) $i=1$, (b) $i=5$, (c) $i=11$, (d) $i=14$.

Walsh-Hadamard sequences with length 16 together with their autocorrelation functions are shown on Fig. 3. First sequence (Fig. 3a) $WHS(1,16)$ contains all ones. There is no state transition in the sequence and its autocorrelation function is the same as autocorrelation function of radar pulse without intra-pulse modulation. First WHS for any N has these properties and it is not suitable for binary coding of radar signal. The 5th sequence (Fig.3b) $WHS(5,16)$ has only three state transitions and the shortest subpulse run is 4, so $G_{BB}=6$ dB. The 11th sequence (Fig.3c) $WHS(11,16)$ has six state transitions the shortest subpulse run is 2, so $G_{BB}=9$ dB. The 14th sequence (Fig.3d) $WHS(14,16)$ has the best performance from the all above mentioned. It has thirteen state transitions and the shortest subpulse run is 1, so $G_{BB}=12$ dB.

As the result of WHS property analysis for $N=8$, 16 and 32 we can observe following conclusions:

- The first sequence (all ones) cannot be used for binary encoding of radar signals;
- Other sequences can be divided into 3 groups – the worst, good, the best - according to above mentioned rules;
- All sequences in the group provide approximately the same performance.

3 Pulse Radar Code with WHS In Presence of Active Jamming

For an analysis of using WHS encoded BPSK-IM radar signals in the case of active noise jamming, a radar model was designed and encoded in Matlab environment. This model simulates the generating and processing of the radar signals with the different jamming-to-signal ratio (JSR). The basic parameters of the proposed model are defined in Table 1.

Table 1. Basic parameters for modeling

PW_m	0.2 μ s
PRI	30 μ s
IF carrier frequency	30 MHz
Sampling period	$1/10 f_{max}$
Barker encoded BPSK-IM	Barker code with length of 13
WHS encoded BPSK-IM	WHS with length 8, 16 and 32
Jamming signal	Gaussian noise

The model of radar system with BPSK-IM modulation, active noise jamming and the signal processing is presented in Fig.4. The synchronizer provides timing for the pulse and subpulse triggering. The modulation signal generator generates bipolar binary sequence which is used for BPSK modulation of carrier frequency. The delay circuit randomly generates delay τ of echo signal within the range from 5 to 25 μ s. Delayed signal $s(t+\tau)$ is mixed in additive mixer with noise $N(t)$ that is generated in the noise jamming generator. The result of mixer is an additive mix of delayed signal and noise given by

$$y(t) = s(t + \tau) + N(t) . \quad (6)$$

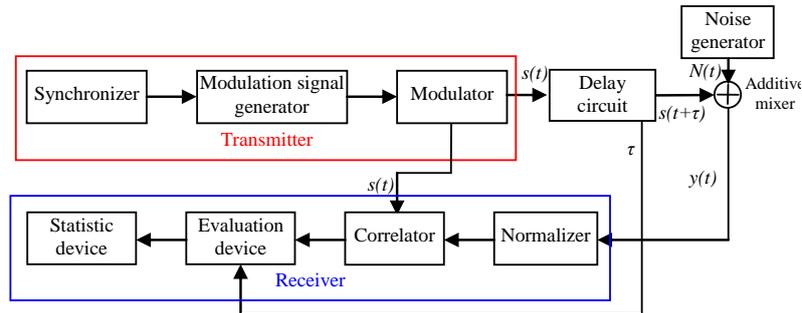


Fig. 4. The model for the binary encoded BPSK-IM signals and jamming generation and processing.

The average power of active noise jamming $N(t)$ was modified during simulation in order to reach power ratio of JSR within the range from -3 to 9 dB. The JSR ratio is expressed by

$$JSR = 10 \log \frac{N(t)}{s(t + \tau)}. \quad (7)$$

The example of the transmitted signal $s(t)$ and delayed echo with additive noise $y(t)$ at the radar receiver input for $JSR = 3\text{dB}$ is shown in Fig. 5.

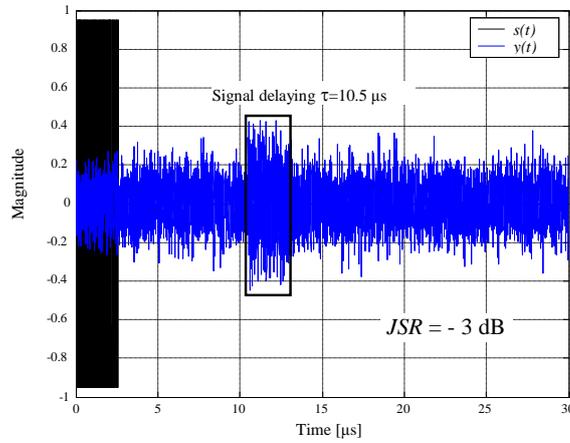


Fig. 5. The transmitted signal and delayed echo with additive noise for $JSR = -3\text{dB}$.

In the next step, the delayed additive mix signal (echo) with noise $y(t)$ is normed and the correlation function with modulated signal $s(t)$ is computed in a correlator according to [13] as

$$cor(m) = \sum_{n=0}^{N-m-1} s_{(n+1)T} \cdot y_{(n+m+1)T}^* \quad (8)$$

where T is a sampling period, n is a number of signal samples, $m = 0, 1, 2, \dots, N-1$ and $*$ represents the complex conjugate. As example, the graph of correlation function $cor(m)$ versus distance for $WHS(14,16)$ with $JSR = 6\text{dB}$ is shown in Fig.6.

A target range is evaluated (in the evaluation device) from the correlation function $cor(m)$ and it is compared with random delay of modulated signal that is adjusted in the delay circuit. A number of correct detection N_{COR} for given JSR is evaluated in the statistic device and after that is computed the probability of detection P_D which is defined as

$$P_D = \frac{N_{COR}}{N_{PRI}} \quad (9)$$

where $N_{PRI} = 10\,000$ is the number of periods in which the statistical evaluation for given JSR was repeated.

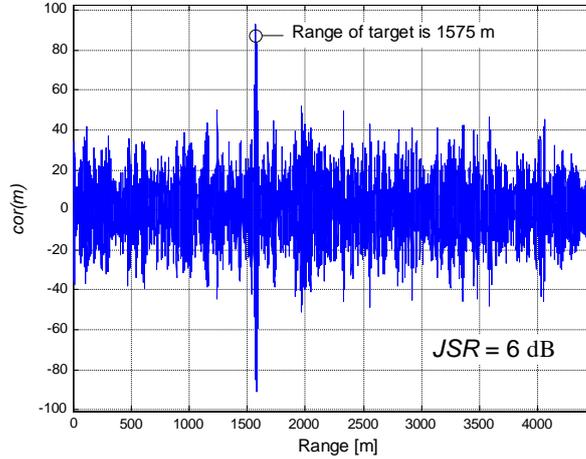


Fig. 6. Graph of correlation function vs. range for WHS(14,16).

4 Results of Numerical Simulations

The simulations were performed to evaluate performance of WHS encoded BPSK-IM radar signals in presence of active noise jamming.

Table 2. Decomposition of WHS for $N=8$ into groups (JSR for $P_D=0,8$)

	The best group		The good group			The worst group	
Row	6	8	2	4	3	7	5
JSR	9,25	9,13	8,74	8,6	8,19	7,57	5,94

Table 3. Decomposition of WHS for $N=16$ into groups (JSR for $P_D=0,8$)

The best group									
Row	14		6	10	16	8	12		
JSR	12,58		12,43	12,41	12,41	12,39	12,25		
The good group							The worst group		
Row	11	15	4	3	2	7	5	13	9
JSR	11,6	11,22	11,11	10,91	10,74	10,32	9,55	8,31	6,27

Decompositions of WHS for $N=8$, 16 and 32 are shown in Table 2, Table 3 and Table 4 respectively together with required JSR for $P_D=0,8$. Decomposition was performed according to the rules described in section 2.3 and according to required JSR to achieve $P_D=0,8$ from simulation results. For each N , $N-1$ sequences (without 1st sequence) were divided into 3 groups – the best group (BG), the good group (GG) and the worst group (WG).

For WSH with N=8 there are 2, 3 and 2 sequences in BG, GG and WG respectively. So we have only 2 sequences with the best performance. To achieve more sequences with the best performance, we have to use WHS with N=16 or 32. For WSH with N=16 there are 6, 6 and 3 sequences in BG, GG and WG respectively. Moreover, for WSH with N=32 there are 10, 10 and 11 sequences in BG, GG and WG respectively.

Table 4. Decomposition of WHS for N=32 into groups (JSR for $P_D=0,8$)

The best group										
Row	30	22	32	24	14	26	10	28	16	12
JSR	15,59	15,57	15,52	15,46	15,38	15,19	15,18	15,13	15,11	15,09
The good group										
Row	6	8	20	27	18	19	11	15	31	23
JSR	14,69	14,67	14,63	14,57	14,56	14,37	14,21	14,02	13,98	13,65
The worst group										
Row	4	3	7	5; 21	2	29	13	9	25	17
JSR	12,89	12,88	12,55	12,34	11,85	11,7	10,99	9,83	8,36	6,13

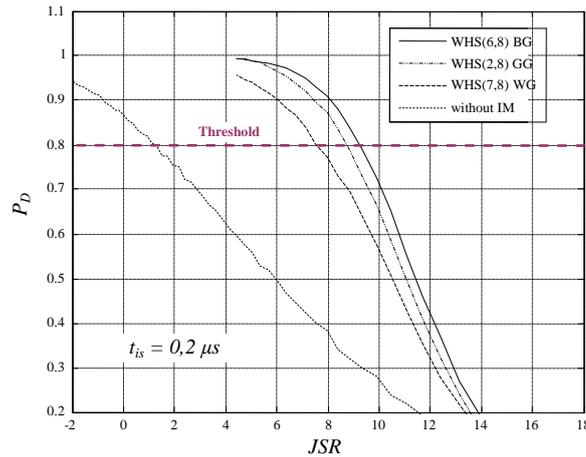


Fig. 7. Probability of correct range detection versus JSR for WHS with length 8.

We can see for the tables above that with using longer sequence length we gain not only more sequences in BG but also better resistance against to the active noise jamming. By increasing length of sequence by 2 we can achieve improvement about 3 dB from the JSR point of view for given P_D .

Relations between probability of correct range detection and JSR for the best sequences from each group (BG, GG, WG) for WHS with length N=8, 16 and 32 are shown in Fig.7, Fig.8 and Fig.9 respectively. In the figures there is also shown the dependence for radar signal without IM.

We can see that even using of sequences from WG for all N provides the significant gain with comparison to system without IM.

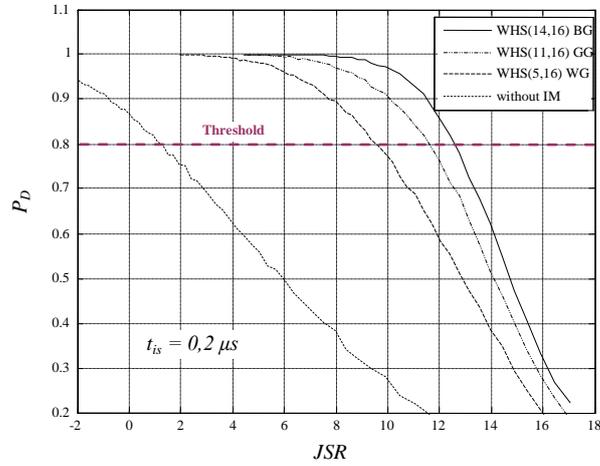


Fig. 8. Probability of correct range detection versus JSR for WHS with length 16.

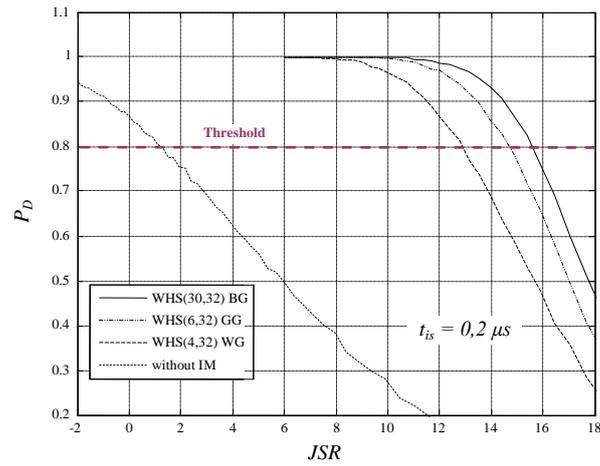


Fig. 9. Probability of correct range detection versus JSR for WHS with length 32.

Comparison of proposed binary encoding of radar signals with using WHS with cases when Barker code of length 13 is used for binary encoding of radar signals and when widely used linear frequency modulation inside pulse duration is used to spread spectrum of radar signals is shown in Fig.10. The better performance than the Barker 13 binary encoded and linear frequency modulation inside pulse duration radar signals can be achieved only by binary encoding by WHS with length 16 or longer. When using WHS with length 16, even by using of sequences from GG we can achieve approximately the same performance as with Barker 13 encoding. Moreover, for WHS with length 32, most of sequences from WG provide approximately the same performance as with Barker 13 encoding.

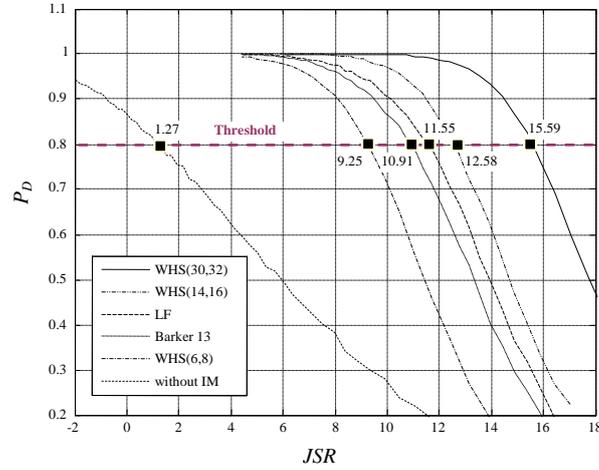


Fig. 10. Comparison of the probability of correct range detection versus JSR for the best sequence from the best group of WHS with length 8, 16 and 32 with the Barker 13 binary encoded and linear frequency modulation inside pulse duration radar signals.

5 Conclusions

The aim of this article was to make an analysis of Walsh-Hadamard sequences for binary encoding of radar signals in presence of the active noise jamming. The decomposition of WHS with length 8, 16 and 32 into 3 groups with approximately the same performance was proposed. According to simulation results the best groups for $N=16$ and 32 will always provide the significantly better performance than systems with Barker 13 binary encoded or linear frequency modulation inside pulse duration radar signals. When using WHS with length 16, even by using of sequences from the good group or WHS with length 32, most of sequences from the worst group provide approximately the same performance as with Barker 13 encoded or linear frequency modulation inside pulse duration radar signals.

The big advantage of using of WHS for binary encoding of radar signals is that we have a set of orthogonal sequences for binary encoding instead of only one sequence in the case of Barker 13. Thus, we can randomly use various sequences in subsequent PRI and to make the system resistant also against the active response jamming, because only echo signals which are coherent with used orthogonal sequence will be processed in receiver. Another advantage of using WHS for binary coding is possibility to avoid mutual interferences of own radar systems. For each radar system only one sequence will be assigned this case.

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